MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

METHODS FOR ADVANCED MATHEMATICS, C3

Practice Paper C3-A

Additional materials: Answer booklet/paper
Graph paper
List of formulae (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72.**
- **You are reminded of the need for clear presentation in your answers.**

Section A (36 marks)

1 Prove that the product of consecutive integers is always even. [2]

2 Find $\dfrac{dy}{dx}$ when $y = \sqrt{1+x^3}$. [3]

3 The graph shows part of the function $y = a\ln(bx)$.
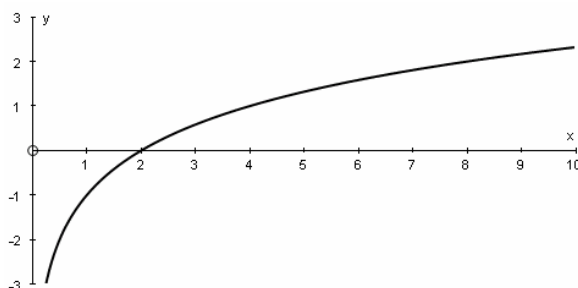
The graph passes through the points $(2, 0)$ and $(4, 1)$.

(i) Show that $b = \dfrac{1}{2}$ and find the exact value of a. [3]

(ii) Solve the inequality $|a\ln(bx)| < 2$. [4]

4 (i) Show that $y = axe^{-x}$ for $a > 0$ has only one stationary point for all values of x.
Determine whether this stationary value is a maximum or minimum point. [5]

(ii) Sketch the curve. [2]

5 Find $\displaystyle\int_2^3 xe^{2x}\,dx$, giving your answer to 1 decimal place. [5]

6 Find $\dfrac{d}{dx}(x\ln x)$ and hence or otherwise find the value of $\displaystyle\int_2^3 \ln x\,dx$, giving your answer in the form $\ln a + b$, where a and b are to be determined. [6]

7   Two quantities, x and θ, vary with time and are related by the equation $x = 5\sin\theta - 4\cos\theta$.

   (i)   Find the value of x when $\theta = \dfrac{\pi}{2}$.   [1]

   (ii)   When $\theta = \dfrac{\pi}{2}$, its rate of increase (in suitable units) is given by $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = 0.1$.

       Show that at that moment $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 0.4$.   [5]

Section B (36 marks)

8   You are given that $\mathrm{f}(x) = \dfrac{x}{x^2 + 1}$ for all real values of x.

   (i)   Show that $\mathrm{f}'(x) = \dfrac{1 - x^2}{\left(x^2 + 1\right)^2}$.   [3]

   (ii)   Hence show that there is a stationary value at $\left(1, \dfrac{1}{2}\right)$ and find the coordinates of the
       other stationary point.   [2]
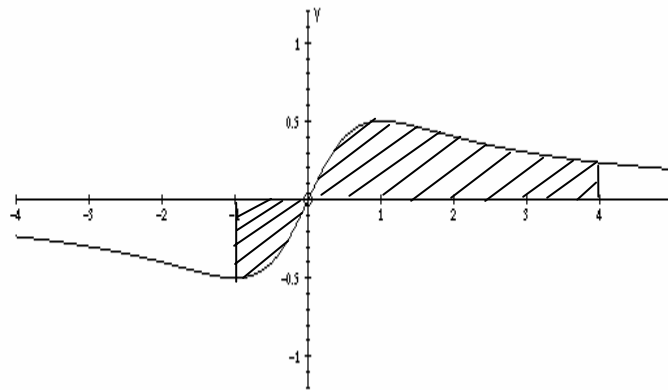
   (iii)   The graph of the curve is shown in Fig. 8.

Fig. 8

       State whether the curve is odd or even and prove the result algebraically.   [2]

   (iv)   Show that $\displaystyle\int_{1}^{4} \dfrac{x}{x^2 + 1}\,\mathrm{d}x = \int_{a}^{b} k\,\dfrac{1}{u + 1}\,\mathrm{d}u$, where the values of a, b and k are to be determined.

                                                                                                                    [5]

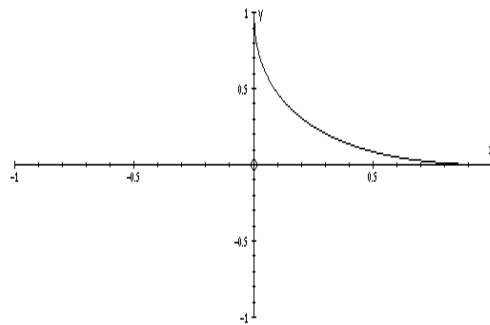   **(v)**   Hence find the area of the shaded region in Fig. 8.   [6]

9 The curve in Fig. 9.1 has equation $\sqrt{x} + \sqrt{y} = 1$.

Fig. 9.1

(i) Show that this is part, but not all of the curve $y = 1 - 2\sqrt{x} + x$.

Sketch the full curve $y = 1 - 2\sqrt{x} + x$. [7]

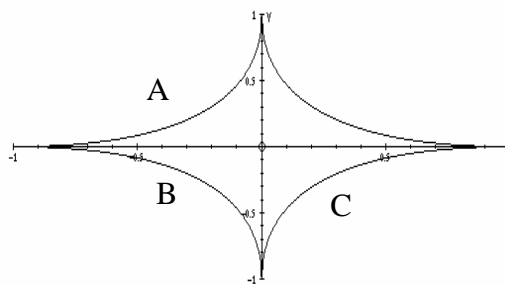**(ii)** Fig.9.2 shows a star shape made up of four parts, one of which is given in part (i) above.

Fig. 9.2

For each of the sections of the shape labelled A, B and C, state the equation of the curve and the domain. [6]

(iii) The shape shown in Fig.9.2 is made into that in Fig. 10.3 by stretching the part of the figure for which $y > 0$ by a scale factor of 2.
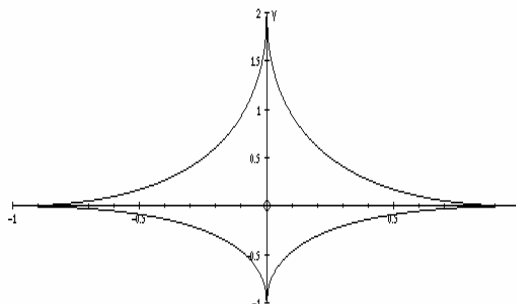

Fig. 9.3

Find the area of this shape. [5]